Indeterminacy with No-Income-Effect Preferences and Sector-Specific Externalities

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Abstract

We examine a two-sector real business cycle model with sector-specifc externalities in the production of distinct consumption and investment goods. In addition, the household utility is postulated to exhibit no income effect on the demand for leisure. Unlike in the one-sector counterpart, we show that equilibrium indeterminacy can result with sufficiently high returns-to-scale in the production of investment goods. We also find that the lower the labor supply elasticity, the lower the threshold level of returns-to-scale needed for generating indeterminacy and sunspots. This finding turns out to be exactly the opposite of that in all existing RBC-based indeterminacy studies.

Keywords: Indeterminacy, Income Effect, Sector-Specific Externalities.

JEL Classification: E30, E32.

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1 Introduction

Starting with the work of Benhabib and Farmer (1994) and Farmer and Guo (1994), there is now an extensive macroeconomic literature that explores indeterminacy and sunspots in the real business cycle (RBC) model.[1] The original Benhabib-Farmer-Guo one-sector economy, with a Cobb-Douglas production function and sufficiently strong increasing returns, displays multiple equilibria and belief-driven business cycle fluctuations when the separable household utility is logarithmic in consumption and convex in hours worked. More recently, Meng and Yip (2008) and Jaimovich (2008) (hereafter MYJ) have shown that a one-sector RBC model, instead with non-separable preferences, always exhibits saddle-path stability and equilibrium uniqueness when there is no income effect on the demand for leisure, regardless of the degree of aggregate returns-to-scale in production. MYJ's result illustrates the critical importance of the income effect associated with the household's labor supply decision in generating indeterminacy and sunspots within one-sector RBC models.

In this paper, we build upon MYJ's analyses and examine the quantitative interrelations between equilibrium indeterminacy and the no-income-effect utility function in a two-sector RBC model. Consumption and investment goods are produced with sector-specific productive externalities *a la* Benhabib and Farmer (1996) and Harrison (2001). Our main findings are summarized as follows. First, unlike in MYJ's one-sector model, our two-sector economy may exhibit an indeterminate steady state, and thus a continuum of perfect-foresight equilibria, when sufficiently strong externalities are present. Intuitively, in order for equilibrium indeterminacy to occur in any dynamic general equilibrium macroeconomic model, the associated first-order conditions must continue to hold when there is a change in non-fundamental expectations. In particular, the household's intertemporal euler equation equates the shadow value of capital sacrificed this period in order to consume – this period's marginal utility of consumption – to the discounted utility value of capital gained next period – its gross rate of return weighted by the marginal utility of next period's consumption. Therefore, upon the expectation of a higher return on capital in the future, agents will

[1] See Benhabib and Farmer (1999) for an excellent survey. With the noted exceptions of Benhabib and Nishimura (1998), and Benhabib, Meng and Nishimura (2000), most studies in this literature postulate constant returns-to-scale at the individual firm level. We also maintain this assumption throughout our analysis.

consume and work more next period. In the MYJ economy, this expectation cannot be self-fulfilled, because any increase in labor hours that may raise the gross return on capital will actually decrease its net return by reducing the marginal utility of consumption. In our model, however, next period's net return on capital also depends positively on its price, which increases when future consumption rises. Therefore, the same increase in hours worked can in fact lead to the expected hike in the return on capital.

Second, we find that a necessary and sufficient condition for indeterminacy and sunspots in our model is sufficiently strong increasing returns-to-scale in the investment sector. The intuition for this result is the same as in the separable preference set-up of Harrison (2001): when agents anticipate the return on capital to increase tomorrow, they need incentive to give up consumption today for more capital accumulation. As long as they will be rewarded with productive investment, in the form of sufficient increasing returns in that sector, it will be worthwhile for them to do so.

Third, and perhaps most surprising, the degree of increasing returns-to-scale required for our model to exhibit multiple equilibria increases with the household's labor supply elasticity. In other words, the lower the labor supply elasticity, the lower the threshold level of investment externalities needed to produce equilibrium indeterminacy. This finding is exactly the opposite of that in all existing RBC-based indeterminacy studies, where an infinitely elastic labor supply is often adopted. The reason is that, to fulfill agents' optimistic expectations in our model economy, and satisfy the household's intertemporal euler equation, movements in labor hours across time periods must be kept small. A large increase in hours worked can reduce the overall future return on capital by reducing next period's marginal utility of consumption. Therefore, the smaller the labor supply elasticity, the easier indeterminacy and sunspots are to obtain, in that lower returns-to-scale in production are needed. In a calibrated version of our model, we find that equilibrium indeterminacy is rendered impossible when the labor supply elasticity is higher than 2.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 examines the model's local stability properties and presents quantitative results. Section 4 concludes.

2 The Economy

Our model incorporates a no-income-effect preference into the discrete-time two-sector real business cycle (RBC) model as in Harrison (2001). Households live forever, and derive utility from consumption and leisure. The production side of the economy consists of two sectors, consumption and investment. For expositional simplicity, firms in each sector produce output using identical technologies, but subject to distinct sector-specific external effects. We assume that there are no fundamental uncertainties present in the economy.

2.1 Firms

In the consumption sector, output is produced by competitive firms using the following technology:

$$Y_{ct} = A_t K_{ct}^{\alpha} L_{ct}^{1-\alpha},\ 0 < \alpha < 1, \tag{1}$$

where K_{ct} and L_{ct} are the capital and labor inputs used in the production of consumption goods. In addition, A_t represents productive externalities that each individual firm takes as given, and is specified as

$$A_t = \left[\bar{K}_{ct}^{\alpha} \bar{L}_{ct}^{1-\alpha}\right]^{\theta_c},\ \theta_c \geq 0, \tag{2}$$

where $\bar{K}_{ct}$ and $\bar{L}_{ct}$ denote the economy-wide average capital and labor used in producing the consumption good, and θ_c measures the degree of sector-specific externalities in the consumption sector.

Similarly, investment goods are produced by competitive firms using the technology

$$Y_{It} = B_t K_{It}^{\alpha} L_{It}^{1-\alpha},\ \text{where}\ \ B_t = \left[\bar{K}_{It}^{\alpha} \bar{L}_{It}^{1-\alpha}\right]^{\theta_I}. \tag{3}$$

Here, K_{It} and L_{It} are capital and hours worked in the investment sector, and B_t represents a productive externality that is an increasing function of the economy-wide average levels of productive capital and labor devoted to producing investment goods. As in Harrison (2001), the degree of sector-specific externalities in the investment sector, denoted as θ_I, is allowed to differ from that for consumption θ_c.

Under the assumptions that factor markets are perfectly competitive and that capital and labor inputs are perfectly mobile across the two sectors, the first-order conditions for the firms' profit maximization problems are

$$r_t = \frac{\alpha Y_{ct}}{K_{ct}} = p_t \frac{\alpha Y_{It}}{K_{It}}, \tag{4}$$

$$w_t = \frac{(1-\alpha)\, Y_{ct}}{L_{ct}} = p_t \frac{(1-\alpha)\, Y_{It}}{L_{It}}, \tag{5}$$

where r_t is the rental rate of capital, w_t is the real wage rate, and p_t denotes the price of investment relative to consumption goods.

2.2 Households

There is a unit measure of identical infinitely-lived households, each with one unit of time endowment and maximizes its present discounted lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t \left[\log(C_t - \frac{\Lambda L_t^{1+\chi}}{1+\chi}) \right], \ 0 < \beta < 1\,, \ \chi \geq 0 \quad \text{and} \ \ \Lambda > 0, \tag{6}$$

where C_t and L_t are the representative household's consumption and hours worked, β is the discount factor, and χ is the inverse of the wage elasticity of labor supply. The budget constraint faced by the representative household is

$$C_t + p_t I_t = Y_t = r_t K_t + w_t L_t, \tag{7}$$

where I_t is gross investment, Y_t is GDP and K_t is the household's capital stock. The law of motion for the capital stock is given by

$$K_{t+1} = (1-\delta) K_t + I_t, \ K_0 \text{ given}, \tag{8}$$

where $\delta \in (0,1)$ is the capital depreciation rate.

The first-order conditions for the household's optimization problem are

$$\Lambda L_t^{\chi} = w_t, \tag{9}$$

$$C_{t+1} - \frac{\Lambda L_{t+1}^{1+\chi}}{1+\chi} = \beta [C_t - \frac{\Lambda L_t^{1+\chi}}{1+\chi}] \left[\frac{r_{t+1} + (1-\delta) p_{t+1}}{p_t} \right] \tag{10}$$

$$\lim_{t \to \infty} \beta^t \frac{K_{t+1}}{C_t} = 0, \tag{11}$$

where (9) equates the slope of the household's indifference curve to the real wage. Note that this equation illustrates the lack of income effect associated with the household's labor supply decision, as C_t is missing. It follows that the income elasticity of intertemporal substitution in hours worked (or leisure) is zero. Moreover, (10) is the standard consumption Euler equation, and (11) is the transversality condition.

Our goal is to examine the local stability properties of the steady state of the above model economy. Before proceeding further, it is useful to obtain an understanding of the implications of incorporating the no-income-effect utility function. In particular, Benhabib and Farmer (1994, 1996), Farmer and Guo (1994) and Harrison (2001), among many others, adopt standard separable preferences

$$U(C_t, L_t) = \log(C_t) - \frac{AL_t^{1+\chi}}{1+\chi}, \quad \chi \geq 0 \quad \text{and } A > 0; \tag{12}$$

and the resulting first-order condition for the choice of labor supply is

$$AL_t^{\chi} = \frac{w_t}{C_t}. \tag{13}$$

Here, the income and substitution effects can be separated. An increase in the real wage, holding consumption constant, will raise L_t – the substitution effect. An increase in consumption, however, leads to a fall in L_t – the income effect. In all RBC-based models, the wage and consumption are procyclical: they move together with output. In addition, in these models, the substitution effect dominates, so that when w_t and C_t increase, so does L_t.

In our model, the first-order condition for hours worked is equation (9), where there is no income effect to counteract the substitution effect. As a result, when the wage (and consumption) increases, L_t rises by more than that in (13).

2.3 Equilibrium and Local Dynamics

We focus on symmetric perfect-foresight equilibria which consist of a set of prices $\{p_t, r_t, w_t\}_{t=0}^{\infty}$ and quantities $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ that satisfies the household's and firms' first-order conditions. Moreover, the aggregate consistency condition requires that $K_{ct} = \bar{K}_{ct}$, $L_{ct} = \bar{L}_{ct}$, $K_{It} = \bar{K}_{It}$ and $L_{It} = \bar{L}_{It}$, for all t. The equalities of demand by households and supply by firms in the consumption and investment sectors are given by $C_t = Y_{ct}$ and $I_t = Y_{It}$. Finally, both the capital and labor markets clear

whereby

$$K_{ct} + K_{It} = K_t, \tag{14}$$

$$L_{ct} + L_{It} = L_t. \tag{15}$$

It is straightforward to show that our model possesses a unique interior steady state. We then take log-linear approximations to the equilibrium conditions in a neighborhood of this steady state to obtain the following dynamic system:

$$\begin{bmatrix} \hat{K}_t \\ \hat{p}_t \end{bmatrix} = J \begin{bmatrix} \hat{K}_{t+1} \\ \hat{p}_{t+1} \end{bmatrix}, \ \hat{K}_0 \text{ given}, \tag{16}$$

where hat variables denote percentage deviations from their steady-state values, and J is the Jacobian matrix of partial derivatives of the transformed dynamic system. The model exhibits saddle-path stability when one eigenvalue of J lies inside and the other outside the unit circle. When both eigenvalues are outside the unit circle, the steady state is indeterminate and thus a sink. When both eigenvalues are inside the unit circle, the steady state becomes a totally unstable source.

3 Stability Properties

As discussed earlier, MYJ show that equilibrium indeterminacy cannot result in the one-sector version of the above model. This section quantitatively examines the local stability properties in our two-sector setting. It turns out that the first result clearly distinguishes our model from MYJ.

Result 1: In sharp contrast to MYJ, indeterminate equilibria result in our model with sufficiently high returns-to-scale.

The exact meaning of "sufficiently high" will be made clear below. As demonstrated by MYJ, in the one-sector version of our model, equilibrium indeterminacy is not possible regardless of the degree of aggregate returns-to-scale in production. What is different here? The answer lies in recalling the intratemporal first-order condition with respect to labor supply, (9), and comparing the intertemporal euler equations from the two models. After rearranging (10), ours is

$$\frac{C_{t+1} - \frac{\Lambda L_{t+1}^{1+\chi}}{1+\chi}}{C_t - \frac{\Lambda L_t^{1+\chi}}{1+\chi}} = \beta \left[\frac{r_{t+1} + (1-\delta)p_{t+1}}{p_t} \right], \tag{17}$$

and its counterpart in the one-sector version is

$$\frac{C_{t+1} - \frac{\Lambda L_{t+1}^{1+\chi}}{1+\chi}}{C_t - \frac{\Lambda L_t^{1+\chi}}{1+\chi}} = \beta[r_{t+1} + (1-\delta)]. \tag{18}$$

In order for equilibrium indeterminacy to result in either economy, the relevant euler equation must be satisfied when there is a change in non-fundamental expectations. For example, starting from the steady state, upon an expected increase in the marginal return on capital, agents sacrifice consumption this period in order to invest more today. Therefore, C_t falls while $k_{t+1.}$ increases. This enables agents to consume more in period $t+1 : C_{t+1}$ rises. However, due to the lack of income effect, as seen in (9), L_t remains unchanged, while the increase in k_{t+1} leads to a rise in L_{t+1}, via the firms' labor demand. The latter has two counteracting effects in the euler equations. First, the bigger (smaller) the increase in L_{t+1}, the smaller (bigger) the increase of the left-hand side. Second, the bigger (smaller) the increase in L_{t+1}, the larger (smaller, or a decrease may occur) the rise in the real interest rate, r_{t+1}. In MYJ's one-sector model with (18), these two effects render the equality impossible. With C_t falling and C_{t+1} rising, a large increase in L_{t+1} is needed for r_{t+1} and the right-hand side to rise. But this would then decrease the left-hand side. At the same time, if the increase in L_{t+1} is small, keeping the left-hand side high, r_{t+1} and the right-hand side cannot rise enough.

However, in our two-sector model with (17), movements of productive resources affect the relative price of investment. In particular, as Benhabib and Farmer (1996) and Harrison (2001) have explained, due to the presence of increasing returns-to-scale, the social PPF showing the trade-off between consumption and investment is convex to the origin. This implies that the shifting of resources towards the production of a good raises the marginal product of each factor used in its production, and lowers the price of that good. Hence, upon agents' optimistic expectations, the shift away from consumption in period t lowers the relative price of investment, p_t; and the shift toward consumption in period $t+1$ increases p_{t+1}. It follows that small movements in L_{t+1} that increase the left-hand side of (17) can also lead to a rise in the right-hand side even though the marginal product of capital may not rise (or rise enough).

Our second result is reminiscent of Harrison (2001):

Result 2: A necessary and sufficient condition for equilibrium indeterminacy is a sufficiently high value of θ_I, the degree of productive externality in investment.

The reason is the same as in Harrison (2001). When agents expect the return on capital to increase tomorrow, they need incentive to give up consumption today for more capital accumulation. As long as they will be rewarded with productive investment, in the form of increasing returns in that sector, it will be worthwhile for them to do so. In other words, it is the return on capital that agents care about, and so their expectation of its increase is fulfilled when there are sufficiently high returns-to-scale in production of the capital good.

Result 2 allows us to set $\theta_c = 0$ for the following quantitative analyses. We calibrate the rest of the model economy using parameter values consistent with post-war US data. Each period in the model is taken to be one quarter. As is common in the real business cycle literature, the capital share of national income, α, is chosen to be 0.3; the discount factor, β, is set equal to 0.99; and the capital depreciation rate, δ, is fixed at 0.025. We then examine the model's local stability properties for different combinations of χ and $\theta_{I.}$. In each parametric configuration, the preference parameter Λ is set to ensure that the steady-state labor hours is equal to 1/3. Figure 1 illustrates the local stability properties of our model, and our third result:

Result 3: Below $\chi = 0.5$ (labor supply elasticity above 2), equilibrium indeterminacy is not possible (not shown).[2] However, as long as $\chi \geq 0.5$, a sufficiently high externality in investment results in local indeterminacy. We denote the threshold value of the externality $\theta_I^{\min}$.

The intuition for this result lies in reconsidering (17). Repeating the experiment, starting from the steady state, upon an expected increase in the marginal return on capital, agents will invest more today. C_t and p_t fall; and $k_{t+1.}$, C_{t+1} and p_{t+1} increase. With no income effect, L_t remains unchanged, but w_{t+1} and L_{t+1} increase. The right-hand side of the euler equation, (17), rises. Therefore, the left-hand side must increase equally; and this requires a small increase in L_{t+1}. Looking again at

[2] For $\chi < .5$, the equilibrium can be either determinate or unstable. In particular, for each $\chi < .5$, the model's steady state is a saddle point below a critical value of θ_I, and is a source above it. This critical value of θ_I increases with χ.

(9), we see that the higher the χ (or the lower the labor supply elasticity), the smaller the increase in L_{t+1}. It turns out that for $\chi < 0.5$, the labor supply elasticity is "too high", thus L_{t+1} rises too much. For $\chi \geq 0.5$, and lower labor supply elasticity, a smaller change in L_{t+1} results.

This also helps us to understand our fourth result, which explains the negative slope of the curve in Figure 1:

Result 4: Equilibrium indeterminacy becomes easier to obtain, in the sense that lower investment externalities are needed, as the labor supply elasticity falls. That is, when χ increases, $\theta_I^{\min}$ decreases $\left(\frac{\partial \theta_I^{\min}}{\partial \chi} < 0\right)$.

This result is exactly the opposite of that in every other RBC-based indeterminacy model.[3] What it means is that as labor supply becomes less elastic, lower returns-to-scale are required for equilibrium indeterminacy. Intuitively, just like with χ, the change in L_{t+1} falls as θ_I increases. This is because higher returns-to-scale imply that the same output gain, and increase in the real wage, can be achieved with a smaller increase in inputs. Therefore, since increases in both χ and θ_I lead to smaller changes in L_{t+1}, the higher the χ, the lower the returns-to-scale needed to keep the increase in L_{t+1} small enough to satisfy (17).

4 Conclusion

This paper extends MYJ's analyses, and examines the stability effect of incorporating no-income-effect preferences into a two-sector real business cycle model with sector-specific externalities. While indeterminacy and sunspots are impossible in the MYJ one-sector economy, their result is overturned here because movements of factors of production affect the relative price of investment in our two-sector setting. In addition, due to the non-separability of consumption and leisure in the household's utility, the key to generating equilibrium indeterminacy is small movements in labor supply. Hence, indeterminacy and sunspots are easier to obtain with a lower labor supply elasticity. In terms of possible extensions, it would be worthwhile to examine the robustness of our results under a generalized non-separable utility *a la* Jaimovich

[3] For example, Benhabib and Farmer (1996, p 433), demonstrate the positive relationship between χ and $\theta_I^{\min}$ in a two-sector model with the separable utility function (12).

(2008) that allows for different degrees of income effect on the demand for leisure. We plan to pursue this research project in the future.

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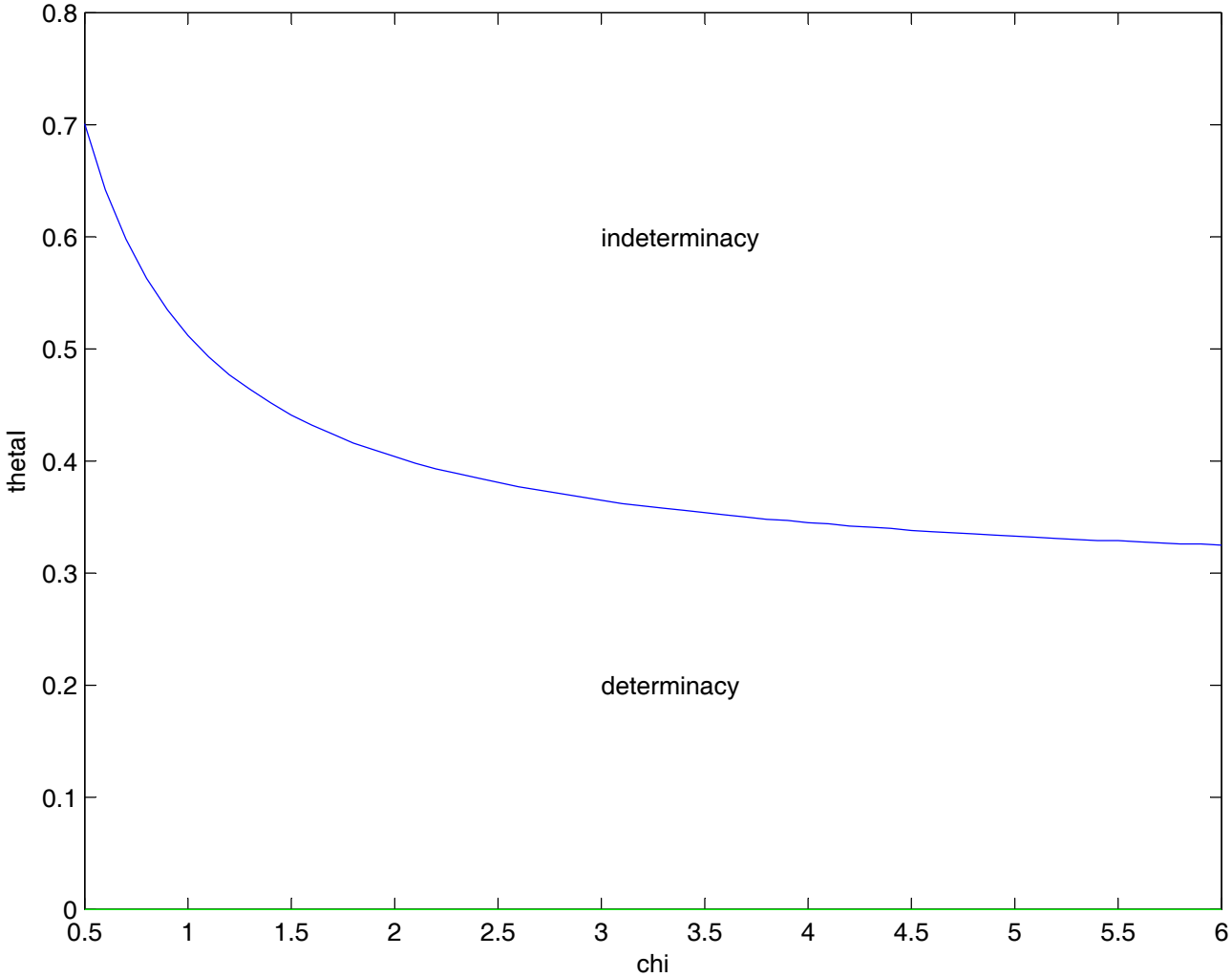
Figure 1: Stability Properties