Useful Government Spending and Macroeconomic (In)stability under Balanced-Budget Rules*

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Abstract

It has been shown that an otherwise standard one-sector real business cycle model may exhibit indeterminacy and sunspots under a balanced-budget rule that consists of fixed and "wasteful" government spending and proportional income taxation. However, the economy always displays saddle-path stability and equilibrium uniqueness if the government finances endogenous public expenditures with a constant income tax rate. In this paper, we allow for productive or utility-generating government purchases in either of these specifications. It turns out that the previous indeterminacy results remain unchanged by the inclusion of useful government spending. By contrast, the earlier determinacy results are overturned when public expenditures generate sufficiently strong production or consumption externalities. Our analysis thus illustrates that a balanced-budget policy recommendation which limits the government's ability to change tax rates does not necessarily stabilize the economy against belief-driven business cycle fluctuations.

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1 Introduction

Recently, there has been growing interest in exploring the interrelations between (local) stability of competitive equilibria and a balanced-budget fiscal policy in the context of standard one-sector real business cycle (RBC) models characterized by perfect competition and constant returns-to-scale in production. This is an important research topic not only for its theoretical relevance, but also for its broad implications for the design, implementation and evaluation of stabilization fiscal policies. In the existing literature, Schmitt-Grohé and Uribe (SU, 1997) show that when the balanced-budget rule consists of fixed government spending and proportional taxation on labor or total income, the economy may exhibit an indeterminate steady state and a continuum of stationary sunspot equilibria. With this type of balanced-budget formulation, when agents' optimism leads to higher investment and hours worked, the government is forced to lower the tax rates as total output rises. This countercyclical tax policy helps fulfill agents' initial optimistic expectations, thus generating equilibrium indeterminacy and belief-driven fluctuations. By contrast, Guo and Harrison (GH, 2004) find that equilibrium indeterminacy disappears under a balanced-budget specification with endogenous government spending and/or transfers financed by separate constant tax rates on labor and capital income. In this case, constant tax rates together with diminishing marginal products of capital and labor dampen the higher anticipated rate of return to investment, which in turn prevent agents' expectations from becoming self-fulfilling. It follows that the economy always exhibits saddle-path stability and equilibrium uniqueness.[1]

In the above-cited work and other related studies, government purchases are assumed to be "wasteful" in that they do not contribute to production or utility.[2] However, the assumption of unproductive/useless public spending, although commonly adopted by academic researchers, is not necessarily the most appealing — at least for developed countries. Motivated by this gap in the literature, we systematically examine the equilibrium impact of "useful" public spending on the economy's stability properties within an otherwise standard one-sector RBC model subject to a balanced-budget constraint. Specifically, government spending may enter the firm's Cobb-Douglas production technology as an input that is complementary to private

[1] See the survey of Benhabib and Farmer (1999) for other mechanisms that generate indeterminacy and sunspots in various versions of real business cycle models.

[2] See, for example, Rudanko (2002), Pintus (2003), Koskela and Puhakka (2005), and Giannitsarou (2006), among others.

capital and labor services. Alternatively, public expenditures may enter the household's utility function as a positive preference externality, either separably or non-separably from private consumption. In each case, without loss of generality, public spending is financed by a single proportional tax rate applied to total income; and the government is required to balance its budget each period according to either Schmitt-Grohé and Uribe's (1997) or Guo and Harrison's (2004) formulation.

Quite interestingly, it turns out that Schmitt-Grohé and Uribe's (1997) indeterminacy results *are* robust to incorporating useful government purchases of goods and services, regardless of how they are introduced to the model. Under their balanced-budget specification, fixed public spending acts simply as a scaling constant in either the firm's production or the household's utility function. It follows that none of the model's stability analysis is affected by allowing for productive or utility-generating government expenditures. This robustness finding highlights the importance of a decreasing income tax schedule for producing indeterminacy and sunspots in Schmitt-Grohé and Uribe's (1997) model.

On the contrary, our analysis shows that Guo and Harrison's (2004) determinacy results are *not* robust to allowing for useful government purchases of goods and services. In particular, when public spending generates sufficiently strong production or consumption externalities, the economy may exhibit indeterminacy and sunspots. We find that when the firm's production function displays private constant returns-to-scale with government spending as an externality, the Guo-Harrison model becomes qualitatively equivalent to Benhabib and Farmer's (1994) laissez-faire economy with an aggregate increasing returns-to-scale technology. As a result, the necessary and sufficient condition for local indeterminacy is identical to that in the Benhabib-Farmer model. That is, the equilibrium wage-hours locus must be upward sloping, and steeper than the labor supply curve.

We also show that the Guo-Harrison model exhibits indeterminacy and sunspots when government spending enters the household's (non-separable) CRRA Cobb-Douglas preferences as a positive consumption externality that is strong enough to yield increasing returns in the household utility. To understand this result, start the economy from its steady state, and suppose that next period's return on capital is expected to increase. Acting upon this belief, agents will invest more and work harder, thereby producing more future output and consumption. Under Guo and Harrison's (2004) balanced-budget formulation, a higher output

leads to an increase in public expenditures, which in turn generates a further rise in the household's own consumption because private consumption and government purchases are Edgeworth complements. Therefore, the percentage increase in private consumption is higher than that in public spending. To validate agents' optimism as a self-fulfilling equilibrium, we find that increasing returns in the household utility are necessary and sufficient to compensate the above growth difference such that the intertemporal consumption Euler equation continues to hold.

On the other hand, Guo and Harrison's (2004) determinacy results remain unchanged when agents' preferences display additive separability between private and public consumption goods. In this case, the marginal utility of the representative household's own consumption is independent of government purchases. As a result, the inclusion of utility-generating public spending has no impact on the model's equilibrium conditions and stability properties.

In terms of policy implications, the results of Schmitt-Grohé and Uribe (1997) and Guo and Harrison (2004) together suggest that if the government intends to stabilize the economy against business cycles driven by agents' animal spirits, a balanced-budget rule that restricts the fiscal authority's ability to change tax rates is called for. However, our analysis in this paper illustrates that such a policy recommendation is not necessarily valid if government purchases of goods and services are useful. Specifically, even under a fixed income tax rate, a one-sector RBC model may exhibit indeterminacy and sunspots when public spending serves as a complementary input in firms' production or provides utility non-separably from the household's own consumption.

The remainder of the paper is organized as follows. Section 2 describes the model economy. Section 3 analyzes the stability effects of useful government spending under the SU or GH balanced-budget requirement. Section 4 concludes.

2 The Model

Our model economy consists of households, firms and the government. In particular, we consider three specifications that incorporate useful government spending into an otherwise standard one-sector real business cycle (RBC) model under a balanced-budget fiscal policy. First, it may enter the firm's problem as a factor of production. Second and third, it may enter the household's utility function as a positive preference externality, which can be either

separable or non-separable from private consumption. We then examine two balanced-budget formulations for each case: one in which the government chooses a countercyclical income tax rate with fixed public spending (SU); and the other in which the proportional income tax rate is a constant and government expenditures change over time (GH). All six possibilities are nested in the following model.

2.1 Households

The economy is populated by a unit measure of identical infinitely-lived households. Each household is endowed with one unit of time and maximizes

$$\int_0^\infty e^{-\rho t}[U(C_t, G_t) - AH_t]dt, \ 0 < \rho < 1 \text{ and } A > 0, \tag{1}$$

where ρ is the subjective discount rate, and C_t and H_t are the individual household's consumption and hours worked, respectively. G_t denotes *aggregate* government purchases of goods and services that are determined outside the household's control. The function $U(\cdot)$ is increasing and strictly concave with respect to private consumption C_t, and also non-decreasing in public expenditures G_t, indicating the possibility of a positive preference externality. In addition, we postulate that $U(\cdot)$ may or may not exhibit additive separability between C_t and G_t. The linearity of (1) in hours worked draws on the formulation of indivisible labor described by Hansen (1985) and Rogerson (1988). Finally, we assume that there are no fundamental uncertainties present in the economy.

The budget constraint faced by the representative household is given by

$$\dot{K}_t = (1 - \tau_t)(w_t H_t + r_t K_t) - \delta K_t - C_t, \ K_0 > 0 \text{ given}, \tag{2}$$

where K_t is the household's capital stock, w_t is the real wage, r_t is the rental rate of capital, $\delta \in (0, 1)$ is the capital depreciation rate, and τ_t is the proportional income tax rate.[3] We require that $\tau_t \geq 0$ to rule out the existence of income subsidies that could only be financed by lump-sum taxation. We also require that $\tau_t < 1$ so that households have incentive to supply labor and capital services to firms.

[3] All of the results in this paper are robust to allowing for separate tax rates on capital and labor income and/or the existence of lump-sum transfers. Hence, without loss of generality, we adopt the simplest fiscal specification: a single proportional income tax rate and no lump-sum transfers.

2.2 Firms

There is a continuum of identical competitive firms, with the total number normalized to one. The representative firm produces output Y_t, using capital, labor and aggregate government spending as complementary inputs, with the following Cobb-Douglas production function (Barro, 1990):

$$Y_t = K_t^{\alpha} H_t^{1-\alpha} G_t^{\eta}, \;\; 0 < \alpha < 1, \;\; \eta \geq 0. \tag{3}$$

Notice that the technology (3) exhibits constant returns-to-scale with respect to private capital and labor inputs; and η captures the degree of a positive external effect that public spending exerts on the firm's production. Moreover, we assume that $\eta < 1-\alpha$ to rule out the possibility of sustained endogenous growth. Under the assumption that factor markets are perfectly competitive, firms take G_t as given and maximize their profits according to

$$r_t = \alpha \frac{Y_t}{K_t}, \tag{4}$$

$$w_t = (1-\alpha) \frac{Y_t}{H_t}, \tag{5}$$

where α and $1-\alpha$ represent the capital and labor share of national income, respectively.

2.3 Government

The government purchases goods and services, and balances its budget each period. Therefore, the government's instantaneous budget constraint is given by

$$G_t = \tau_t Y_t, \tag{6}$$

which becomes $G = \tau_t Y_t$ in Schmitt-Grohé and Uribe's (1997) model, and $G_t = \tau Y_t$ in Guo and Harrison's (2004) economy. Finally, combining equations (2) and (6) yields the aggregate resource constraint for the economy

$$C_t + \dot{K}_t + \delta K_t + G_t = Y_t. \tag{7}$$

3 Macroeconomic Stability and Instability

This section examines stability of equilibria in six versions of the above economy. First, we allow public spending to enter the firm's production function, but not the household's utility. Then, government expenditures are postulated to enter the household utility as a positive preference externality, either separably or non-separably with private consumption, but not the firm's production technology. It turns out that Schmitt-Grohé and Uribe's (1997) indeterminacy results are robust to useful government spending, regardless of how it is introduced to the model. By contrast, Guo and Harrison's (2004) determinacy results may be reversed when public expenditures serve as a complementary input in firms' production or provide utility non-separably from the household's own consumption. This implies that, contrary to the existing policy recommendation, a balanced-budget rule with constant tax rates does not necessarily eliminate the possibility of expectations-driven business cycle fluctuations.

3.1 Productive Government Spending

In the first two cases under examination, the firm's production function is given by (3) with $\eta > 0$, but G_t is left out of the household utility (1). Specifically, as in Schmitt-Grohé and Uribe (1997), the momentary preferences are given by

$$\log C_t - AH_t, \tag{8}$$

which implies that the household's labor supply curve is horizontal with a slope equal to zero.

Under the SU balanced-budget formation $G = \tau_t Y_t$, the firm's production technology becomes

$$Y_t = G^{\eta} K_t^{\alpha} H_t^{1-\alpha}. \tag{9}$$

Therefore, public spending acts simply as a scaling constant in private production. It follows that the model's local stability properties are exactly the same as those in Schmitt-Grohé and Uribe (1997): countercyclical income taxation may result in equilibrium indeterminacy and belief-driven business cycle fluctuations. When agents are optimistic and expect a higher return on next period's capital, they will invest more and work harder, leading to a higher total output. Given fixed government expenditures, the tax rate must fall in order for the government to balance its budget. As a result, the after-tax return on next period's capital may rise, thus validating agents' initial optimistic expectations.

Next, we consider the GH specification with $G_t = \tau Y_t$. Substituting this balanced-budget constraint into (3) yields

$$Y_t = \tau^{\frac{\eta}{1-\eta}} K_t^{\frac{\alpha}{1-\eta}} H_t^{\frac{1-\alpha}{1-\eta}}, \tag{10}$$

where $\frac{\alpha}{1-\eta} < 1$ because sustained endogenous growth is not allowed in our analysis.[4] Notice that the aggregate production function (10) is identical, to a constant, to that in Benhabib and Farmer's (1994) one-sector indeterminate RBC model under laissez-faire. In this case, it can be shown that the *necessary and sufficient* condition for indeterminacy and sunspots is given by

$$\frac{1-\alpha}{1-\eta} - 1 > 0, \tag{11}$$

which is exactly the same as in the Benhabib-Farmer model.[5] That is, the equilibrium wage-hours locus must be upward sloping, and steeper than the labor supply curve.[6]

In sum, when government spending serves as an input to the firm's production, Schmitt-Grohé and Uribe's (1997) indeterminacy results are not affected, whereas Guo and Harrison's (2004) determinacy results are.[7] Specifically, the Guo-Harrison model with productive public expenditures becomes qualitatively equivalent to Benhabib and Farmer's (1994) laissez-faire economy with aggregate increasing returns-to-scale in production because of positive capital/labor externalities or monopolistic competition.

3.2 Utility-Generating Government Spending

In these specifications, we allow government purchases to provide utility as a positive preference externality, but leave them out of the firm's production function ($\eta = 0$). Hence, output is

[4] By contrast, Palivos, Yip and Zhang (2003) examine the possibility of multiple balanced-growth paths that exhibit global indeterminacy in a one-sector endogenous growth model with productive government spending and the G-H balanced-budget specification.

[5] It is now well known that the required degree of increasing returns-to-scale to satisfy the Benhabib-Farmer condition for local indeterminacy (11) is too high to be empirically plausible (Burnside 1996; Basu and Fernald, 1997). However, subsequent research has shown that in a one-sector RBC model with variable capital utilization (Wen, 1998) or in a two-sector RBC model with sector-specific externalities (Benhabib and Farmer, 1996; Perli, 1998; Weder, 2000; Harrison, 2001), the minimum level of increasing returns needed for equilibrium indeterminacy is much less stringent. To maintain comparability with Schmitt-Grohé and Uribe (1997) and Guo and Harrison (2004), we adopt the one-sector RBC formulation with constant capital utilization.

[6] Substituting the social technology (10) into the logarithmic version of firms' labor demand condition (5) indicates that the slope of the equilibrium wage-hours locus is given by $\frac{1-\alpha}{1-\eta} - 1$. Notice that (11) is a necessary, not sufficient, condition for local indeterminacy in discrete-time one-sector RBC models (Schmitt-Grohé, 1997).

[7] Ortigueira (2002) obtains a similar result in a two-sector endogenous growth model with human capital accumulation, namely productive government spending creates "fiscal increasing returns" that may lead to indeterminate equilibria and expectations-driven fluctuations.

produced by

$$Y_t = K_t^{\alpha} H_t^{1-\alpha}, \;\; 0 < \alpha < 1. \tag{12}$$

Regarding the functional form of $U(C_t, G_t)$, while there are several empirical studies that examine the substitutability of public spending for private consumption, the existing evidence is mixed. For example, Karras (1994) and Amano and Wirjanto (1998) cannot reject the hypothesis of additive separability in C_t and G_t. By contrast, Ni (1995) provides empirical support for the (non-separable) CRRA Cobb-Douglas utility specification and the Edgeworth complementarity between private and public consumption $\frac{\partial U(\cdot)/\partial C_t}{\partial G_t} > 0$.[8] For completeness of the analysis, we investigate both possibilities.

3.2.1 Separable Private and Public Consumption

Motivated by the empirical results of Karras (1994) and Amano and Wirjanto (1998), we first consider a household preference that exhibits additive separability between private consumption and public spending. As an illustrative example, let the momentary utility function be[9]

$$\log C_t + \gamma \log G_t - AH_t, \;\; \gamma > 0. \tag{13}$$

In this case, regardless of the exact formulation of a balanced-budget rule, the marginal utility of C_t is independent of G_t. Therefore, the inclusion of utility-generating government purchases does not have any impact on the model's equilibrium conditions and stability analyses. This implies that Schmitt-Grohé and Uribe's (1997) indeterminacy results and Guo and Harrison's (2004) determinacy results both will remain unchanged when private consumption and public expenditures are additively separable in the household utility.

[8] Ni (1995) considers the linear and Cobb-Douglas formulations of *effective* consumption, which specifies how private consumption and government purchases are combined into a composite good that enters the CRRA-variety $U(C_t, G_t)$. When effective consumption is postulated as a linear function, the Edgeworth complementarity between C_t and G_t implies that $U(\cdot)$ is decreasing in public spending. This violates a standard assumption on the household preferences, and generates more unstable point estimates compared to those under the Cobb-Douglas form of effective consumption. Moreover, empirical results based on the generalized CES formulation of effective consumption indicate that the Cobb-Douglas specification is more appropriate than the linear form. Based on these findings of Ni (1995), we adopt the CRRA Cobb-Douglas preference formulation (equation 14) in our analysis.

[9] This log-log preference specification has been adopted by Guo and Lansing (1999) and Cassou and Lansing (2004), among others.

3.2.2 Non-Separable Private and Public Consumption

Based on the empirical findings of Ni (1995), and to highlight the stability effect of allowing non-separability between private consumption and government expenditures, we consider the following CRRA Cobb-Douglas utility function that is a monotonic transformation of (13):

$$\frac{\left(C_t^{\theta_1} G_t^{\theta_2}\right)^{1-\sigma}}{1-\sigma} - AH_t, \;\; \theta_1, \theta_2, \sigma > 0 \text{ and } \sigma \neq 1, \tag{14}$$

where $0 < \theta_1 (1-\sigma) < 1$ to ensure strict concavity with respect to private consumption, $\theta_2 (1-\sigma) > 0$ to indicate a positive public-spending preference externality, and the parameter σ denotes the inverse of the intertemporal elasticity of substitution in effective consumption $C_t^{\theta_1} G_t^{\theta_2}$.[10] When $\sigma = 1$, the household utility becomes logarithmically separable in private and government consumption, as in (13) with $\gamma = \frac{\theta_2}{\theta_1}$. Moreover, $(\theta_1 + \theta_2)(1-\sigma)$ measures the degree of homogeneity or returns-to-scale in the function $U(C_t, G_t)$. In contrast to the preceding analysis, the marginal utility of private consumption is now increasing in government purchases (*i.e.*, C_t and G_t are Edgeworth complements), which in turn may affect the model's dynamic properties.

Under the SU balanced-budget formation $G = \tau_t Y_t$, the household's momentary utility function becomes

$$\frac{\left(C_t^{\theta_1} G^{\theta_2}\right)^{1-\sigma}}{1-\sigma} - AH_t. \tag{15}$$

Therefore, public spending acts simply as a scaling constant in the household preferences. It follows that the model's local stability properties are identical to those in Schmitt-Grohé and Uribe (1997): a balanced-budget rule with countercyclical income taxes can raise the after-tax return on capital, helping fulfill agents' optimism about the future of the economy.

Next, we consider the GH specification with $G_t = \tau Y_t$. In this case, households take G_t as given, and maximize their discounted lifetime utilities according to the budget constraint (2) together with

$$\frac{A}{\theta_1 C_t^{\theta_1(1-\sigma)-1} G_t^{\theta_2(1-\sigma)}} = (1-\tau)\, w_t, \tag{16}$$

$$\frac{\dot{C}_t}{C_t} = \frac{\theta_2 (1-\sigma)}{1-\theta_1 (1-\sigma)} \frac{\dot{G}_t}{G_t} + \frac{1-\tau}{1-\theta_1 (1-\sigma)} r_t - \frac{\delta + \rho}{1-\theta_1 (1-\sigma)}, \tag{17}$$

[10] Ni (1995) restricts his empirical analysis to the cases in which $\theta_1 + \theta_2 = 1$. By contrast, we relax Ni's restriction by allowing $\theta_1 + \theta_2$ to take any positive value as long as (14) is strictly concave in C_t and also increasing in G_t.

$$\lim_{t\to\infty} e^{-\rho t}\frac{K_t}{\theta_1 C_t^{\theta_1(1-\sigma)-1} G_t^{\theta_2(1-\sigma)}} = 0, \tag{18}$$

where (16) equates the slope of the representative household's indifference curve to the after-tax real wage. In addition, (17) is a modified consumption Euler equation in which the term $\frac{\dot{G}_t}{G_t}$ reflects the impact of public spending on the marginal utility of private consumption, and (18) is the transversality condition.

To facilitate the analysis of perfect-foresight dynamics, we make the following logarithmic transformation of variables: $k_t \equiv \log(K_t), y_t \equiv \log(Y_t)$ and $c_t \equiv \log(C_t)$. With this transformation and using $\frac{\dot{G}_t}{G_t} = \frac{\dot{Y}_t}{Y_t}$ from the GH balanced-budget rule, the two dynamic equations (2) and (17) can be rewritten as

$$\dot{k}_t = (1-\tau)e^{y_t-k_t} - \delta - e^{c_t-k_t},\ k_0 \text{ given}, \tag{19}$$

$$\dot{c}_t = \frac{\theta_2(1-\sigma)}{1-\theta_1(1-\sigma)}\dot{y}_t + \frac{\alpha(1-\tau)}{1-\theta_1(1-\sigma)}e^{y_t-k_t} - \frac{\rho+\delta}{1-\theta_1(1-\sigma)}. \tag{20}$$

For (19) and (20) to be an autonomous pair of differential equations, we need to express y_t in terms of k_t and c_t, and also $\dot{y}_t$ in terms of $\dot{k}_t$ and $\dot{c}_t$. Combining (5), (6), (12) and (16) yields the following expression for y_t:

$$y_t = \lambda_0 + \lambda_1 k_t + \lambda_2 c_t, \tag{21}$$

where

$$\lambda_0 = \frac{(1-\alpha)}{\alpha-\theta_2(1-\alpha)(1-\sigma)}\log\left[\frac{\theta_1(1-\alpha)(1-\tau)\tau^{\theta_2(1-\sigma)}}{A}\right],$$

$$\lambda_1 = \frac{\alpha}{\alpha-\theta_2(1-\alpha)(1-\sigma)} \text{ and } \lambda_2 = \frac{-(1-\alpha)[1-\theta_1(1-\sigma)]}{\alpha-\theta_2(1-\alpha)(1-\sigma)}.$$

Taking the time derivative on both sides of (21) leads to

$$\dot{y}_t = \lambda_1\dot{k}_t + \lambda_2\dot{c}_t. \tag{22}$$

Substituting (21) and (22) into (19) and (20) produces the required pair of autonomous differential equations as follows:

$$\dot{k}_t = (1-\tau)e^{\lambda_0+(\lambda_1-1)k_t+\lambda_2 c_t} - \delta - e^{c_t-k_t},\ k_0 \text{ given}, \tag{23}$$

$$\dot{c}_t = \frac{(1-\tau)\left[\alpha+\lambda_1\theta_2(1-\sigma)\right]}{1-\theta_1(1-\sigma)-\lambda_2\theta_2(1-\sigma)}e^{\lambda_0+(\lambda_1-1)k_t+\lambda_2 c_t} - \frac{\lambda_1\theta_2(1-\sigma)}{1-\theta_1(1-\sigma)-\lambda_2\theta_2(1-\sigma)}e^{c_t-k_t}$$

$$-\left[\frac{1+\lambda_1\theta_2(1-\sigma)}{1-\theta_1(1-\sigma)-\lambda_2\theta_2(1-\sigma)}\right]\delta - \frac{\rho}{1-\theta_1(1-\sigma)-\lambda_2\theta_2(1-\sigma)}. \tag{24}$$

It is straightforward to show that the above autonomous dynamical system possesses a unique interior steady state. We can then compute the Jacobian matrix of (23) and (24) evaluated at the steady state. The trace and determinant of the Jacobian are given by

$$Tr = -(1-\alpha)(1-\tau)x_1 + \left[\frac{1-(\theta_1+\theta_2)(1-\sigma)}{1-\theta_1(1-\sigma)}\right]x_2, \tag{25}$$

and

$$Det = \frac{(1-\alpha)(1-\tau)\left[(\theta_1+\theta_2)(1-\sigma)-1\right]}{1-\theta_1(1-\sigma)}x_1x_2, \tag{26}$$

where

$$x_1 = \frac{\rho+\delta}{\alpha(1-\tau)} > 0 \quad \text{and} \quad x_2 = \frac{\rho+(1-\alpha)\delta}{\alpha} > 0.$$

The local stability of the steady state can be deduced from the above trace and determinant. With one initial condition k_0 given, the steady state is a saddle point when the eigenvalues of the dynamical system (23) and (24) are of opposite sign, hence the determinant (26) is negative. When there is no public-spending preference externality ($\theta_2 = 0$), as in Guo and Harrison (2004), it is straightforward to show that the determinant becomes

$$Det = -(1-\alpha)(1-\tau)x_1x_2 < 0, \tag{27}$$

which implies that the model exhibits saddle-path stability and equilibrium uniqueness. In this case, a constant income tax rate together with diminishing returns to capital and labor inputs will reduce the higher anticipated returns from belief-driven labor and investment spurts, thus preventing agents' optimistic expectations from becoming self-fulfilling. Therefore, as in a standard RBC model under laissez-faire, the economy does not display endogenous business cycles driven by agents' animal spirits.

On the other hand, the steady state is locally indeterminate (a sink) when both eigenvalues of the dynamical system (23) and (24) have negative real parts so that the trace (25) is negative and the determinant (26) is positive. Since $x_1, x_2 > 0, 0 < \alpha, \theta_1(1-\sigma) < 1$ and $0 \leq \tau < 1$, the determinant is positive if and only if

$$(\theta_1 + \theta_2)(1 - \sigma) > 1, \tag{28}$$

which means that the household utility displays increasing returns-to-scale in private consumption and public spending. Moreover, given the first term of (25) is always negative, the trace becomes positive only when $(\theta_1 + \theta_2)(1 - \sigma) < 1$. Therefore, condition (28) not only leads to a positive determinant, but also guarantees a negative trace. This implies that (28) is the *necessary and sufficient* condition for the model to exhibit equilibrium indeterminacy and sunspot-driven fluctuations.[11]

To understand the above indeterminacy result, consider the following consumption Euler equation, in discrete-time for ease of interpretation:

$$\left(\frac{C_{t+1}}{C_t}\right)^{1-\theta_1(1-\sigma)} = \beta \left(\frac{G_{t+1}}{G_t}\right)^{\theta_2(1-\sigma)} \left[(1-\tau)r_{t+1} + 1 - \delta\right], \tag{29}$$

where $\beta = \frac{1}{1+\rho}$ is the discount factor. Start the economy from its steady-state equilibrium at period t; and suppose that agents become optimistic about next period's return on capital.[12] Acting upon this belief, households will invest more (raising K_{t+1}) and work harder (raising H_{t+1}, since capital and labor are complementary inputs in firms' production), thereby producing more output and higher consumption in period $t+1$. This in turn causes the left-hand side of (29) to increase. Under constant returns-to-scale in production, a higher capital stock is associated with a lower rate of return ($\frac{\partial r_{t+1}}{\partial K_{t+1}} < 0$). Without a public-spending preference externality ($\theta_2 = 0$), agents' expectations cannot be self-fulfilled because the right-hand side of (29) will fall. However, under the GH balanced-budget formulation with a constant income tax rate, a larger Y_{t+1} leads to an increase in government purchases of goods and services. When $\theta_2 > 0$, the marginal utility of private consumption rises with public spending; therefore a higher G_{t+1} will generate a further increase in C_{t+1}. It follows that the percentage increase in G_{t+1} is smaller than that in C_{t+1}. To validate agents' initial optimism as a self-fulfilling equilibrium, the right-hand side of (29) needs to rise enough to overcome the fall in r_{t+1} and

[11] This result is reminiscent of Cazzavillan (1996) who examines an one-sector endogenous growth model with inelastic labor supply and government purchases entering both the firm's production and the household's utility functions. As it turns out, the model exhibits multiple balanced growth paths and endogenous growth fluctuations when the household utility displays increasing returns-to-scale in private consumption and public spending. In addition, Zhang (2000) shows that Cazzavillan's model exhibits various forms of local stability properties or Hopf bifurcations when the social technology exhibits increasing returns in private capital and public expenditures.

[12] It can be shown that at the steady state, the consumption-output ratio is given by $\frac{(1-\tau)[\rho+(1-\alpha)\delta]}{\rho+\delta}$, and the public spending-output ratio is equal to τ.

the growth difference between government expenditures and private consumption. It turns out that this requires $\theta_2 (1-\sigma) > 1 - \theta_1 (1-\sigma)$, which is condition (28).[13]

In sum, when government spending enters the household's CRRA Cobb-Douglas preferences as a positive externality and non-separably from private consumption, Schmitt-Grohé and Uribe's (1997) indeterminacy results are unaffected, whereas Guo and Harrison's (2004) determinacy results are affected. Specifically, the Guo-Harrison model exhibits indeterminacy and sunspots when the public-spending preference externality is sufficiently strong to generate increasing returns in the household utility.[14]

4 Conclusion

Schmitt-Grohé and Uribe (1997) and Guo and Harrison (2004) show that the exact formulation of a balanced-budget fiscal policy plays an important role in affecting the local stability properties of an otherwise standard one-sector real business cycle model with wasteful government purchases of goods and services. In this paper, we extend these analyses by allowing public spending to enter either the firm's production or the household's utility function. As it turns out, Schmitt-Grohé and Uribe's (1997) indeterminacy results remain unchanged by the inclusion of useful government spending, regardless of how it is introduced in the model. On the contrary, Guo and Harrison's (2004) determinacy results are reversed when public expenditures generate sufficiently strong production or consumption externalities. In particular, we show in the Guo-Harrison model with productive government spending, the necessary and sufficient condition for indeterminacy and sunspots is identical to that in Benhabib and Farmer's (1994) laissez-faire economy. We also find that equilibrium indeterminacy in the Guo-Harrison model with utility-generating public expenditures requires the household utility to display increasing returns-to-scale in private and public consumption. From a policy standpoint, our

[13] Since θ_1, θ_2, σ, $\theta_2 (1-\sigma) > 0$ and $\theta_1 (1-\sigma) < 1$, satisfying the indeterminacy condition (28) requires that $\theta_1 + \theta_2 > 1$ and $\sigma < 1$. To our knowledge, there is no available empirical evidence that is based on $\theta_1 + \theta_2 \neq 1$. On the other hand, $\sigma < 1$ is not inconsistent with the existing statistical evidence from U.S. time series, as in Eichenbaum, Hansen and Singleton (1988), among others. While beyond the scope of this paper, an examination of the empirical plausibility of condition (28) is a worthwhile topic for future research.

[14] Fernández, Novales and Ruiz (2004) also examine the stability effects of the GH balanced-budget formulation in a one-sector RBC model, but with (non-separable) complementarity between public expenditures and leisure in the household's utility function. In this case, equilibrium indeterminacy occurs if and only if the reduced-form Frisch labor supply curve is downward sloping, and is steeper than the (negatively-sloped) labor demand curve. However, the requisite condition on labor supply exhibits an undesirable implication — leisure is not a normal good. See Aiyagari (1995), Farmer and Guo (1995), Bennett and Farmer (2000), Hintermaier (20003) and Nakajima (2006) for more discussions.

analysis illustrates that in the context of standard one-sector real business cycle models with useful government spending, a balanced-budget rule that restricts the government's ability to change tax rates does not completely rule out the occurrence of endogenous business cycles driven by agents' self-fulfilling beliefs.

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